Trends in Grasping

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Abstract

Research and development in robotics and industrial automation has created a need for good grasp planning algorithms for a variety of object shapes and hand types. This in turn has stimulated research on the inherent computational geometry of grasping. The purpose of this paper is to survey some of the recent grasping results, algorithms, and ideas. We will place particular emphasis on research involving the synthesis of optimal grasps.

We include a brief discussion of a few of the many problems that arise in the context of grasping. The main focus of this paper, however, is on the following problem and its variants: Given a description of an object (e.g. Is the object concave or convex? Is it a polygonal object or is it curved?) and a gripper (e.g. How many fingers/contacts does the gripper have? What type of contacts are they? Point contacts? Is there friction? What positions can the fingers obtain?), find an "optimal" grasp of the object with the gripper or determine that there is no feasible grasp of the object. As we will see, even the notion of optimal will not always be measured by the same metrics, making a general solution difficult if not impossible to find.

1 Introduction

As the first multifingered robot hands began to appear in research laboratories, the design, analysis, and control of such hands became an active area of research. Numerous analytical approaches were proposed for characterizing grasps and modeling the process of manipulation. In addition, there were significant advances in control strategies, tactile sensing, and grasp planning. Yet it seems that even after years of grasping research and results, we are still a long way from building robots with hands that can independently decide how to pick up and manipulate objects to accomplish everyday tasks.

2 History

As early as 1875, studies into grasping were being conducted. Reuleaux (1875) [Reu75] proved that a two-dimensional grasps requires at least four *point contacts*[1] to satisfy *form-closure*[2]. Over a century later in 1978, Lakshminarayana [Lak78] proved that in three-dimensions at least seven point-contacts are required for form-closure grasps. Other early studies of grasping were conducted by Asada and Hanafusa in 1977 [HA77] and later by Asada in 1979 [Asa79]. However, it was not until the mid-Eighties that multifingered hands began to show up in laboratories and the importance of grasping research grew. Within only a few years, grasping papers were appearing in record numbers at almost every robotics and control conference. A few of the more notable papers were Salisbury and Roth [SR83], Baker, Fortune, and Grosse [BFG85], and Kerr and Roth [Ste86]. In all of these papers, the problem of achieving a good, firm grip on an object was found to be the most fundamental issue underlying the design and control of multifingered hands.

3 Multifingered Hands

By the mid-Eighties, multifingered hands were already in use throughout several laboratories around the world including the Utah/MIT Dextrous Hand (1984), the Stanford/JPL Hand (1981), the Okada Hand (1982), and the Asada Hand (1979). These multifingered hands had been developed to extend the performance of the current day robots but were proving to be difficult to control and utilize. Al-

[1] A *point contact* is obtained when there is no friction between the fingertip and the object. In this case, forces can only be applied in the direction normal to the surface of the object. This is different from a point contact with friction. Point contacts are usually assumed frictionless unless stated otherwise.

[2] Form-closure is intuitively a method of defining a "firm grip" on an object when friction is not taken into account.

Dim.	Point Contact	Point w/Fric.	Soft Fing.
2	4	3	3
3	7	4	4

Table 1: Lower bounds on the number of fingers required to grasp an object.

most overnight these hands provided a plethora of new problems for scientists.

One notable early study was a paper by Mishra, Schwartz, and Sharir [MSS87] which began to study the criteria under which an object could be gripped by a multifingered dextrous hand, assuming no *friction*[3] between the object and the fingers. Mishra *et al.* were one of the first to provide an efficient algorithm for synthesizing point contact finger grips without friction for bounded polyhedral/polygonal objects. Their algorithm had linear running time in the number of faces/sides of the polyhedral/polygon. Their work was also of interest for its presentation of algorithms arising in the study of positive linear spaces which was a natural result of grasp analysis since the contacts always exerted strictly positive forces on the object.

Mishra *et al.* concluded that almost any two-dimensional object can be held at equilibrium by at least four point contacts without friction and at least seven point contacts in three-dimensions. These results, arrived at previously by Reuleaux and Lakshminarayana, are reiterated in Table 1 which shows results for a variety of different contacts and dimensions. Mishra *et al.* also discovered that straight lines, circles, and helixes could not be constrained by any number of point contacts. Since we only deal with bounded objects, this means that any surface of revolution is impossible to balance since no torque in the direction of its axis can be resisted by frictionless grasping. However, it's important to note that for all of these special cases where motion cannot be resisted, the object will be constrained to move between visually indistinguishable positions (assuming the object does not have any additional significant features). This observation led to the general conclusion that any two-dimensional (three-dimensional) object can always be held by four (seven) fingers in a frictionless grip which resists any external forces and torques which move the object to a significantly different spatial position.

[3] *Friction* throughout this paper refers to a simple Coulomb friction model between the finger and object.

4 Modeling Human Grasping

Looking for a new direction in grasping research, Cutkosky [Cut89] decided that the grasping models that were currently being used for multifingered hands contained simplifications and assumptions that limited their application to manufacturing environments. Cutkosky, as well as other researchers at the time, saw the need to make more anthropomorphic hands (i.e. hands more closely resembling humans hands).

In order to achieve a better understanding of grasping, he undertook a study of various grasps used by actual machinists during the course of their work. From that study he developed a classification system or taxonomy of grasps, and with the help of an expert system, was able to make rough guesses as to which grasp type should be used given various input constraints. A reproduction of his classification system shown in Figure 1. The classification of grasps into power grasps and precision grasps is not new and was observed by Napier [Nap56] as early as 1956.

Cutkosky's view of grasp planning was as a set of overlapping constraints arising from the task (e.g. forces and motions that must be imparted), the object (e.g the shape, slipperiness, and fragility of the object), and from the hand or gripper (e.g. the maximum grasp force and maximum opening of the fingers). Within these constraints he defined the space of "feasible grasps." For Cutkosky, choosing an optimal grasp meant defining an objective function to optimized subject to the constraints. The main problem was that there was no agreed upon definition of what an "optimal grasp" was, and therefore no method of defining an objective function.

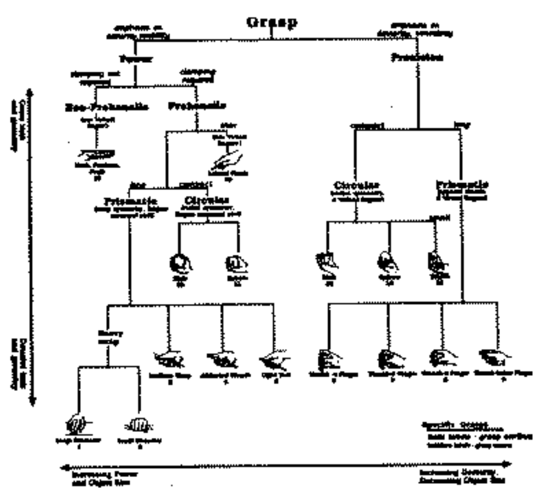

Figure 1: A partial taxonomy of manufacturing grasps. A result of research by Cutkosky in 1986 and 1989.

Even though Cutkosky's expert system was incomplete and never a reliable predictor of how humans would grasp tools or parts, his paper is quite important because it was the first real research of human grasping for robotics. Its major contribution was in helping to raise and clarify additional issues in the study of grasping.

5 Grasping With Uncertainty

In 1988 Brost saw the grasping problem as clouded with issues of uncertainty. Even if grasp planning could be solved, no robot of the time possessed enough accuracy to achieve such an implementation. Robots in the Eighties were hampered by severe accuracy limitations and thus a wealth of new techniques for dealing with these inherent uncertainties had surfaced. Most of the contribution in this area came from Lozano-Pérez, Mason, and Taylor [LMT84] along with related work by Erdmann [Erd86], Peshkin [Pes90], Whitney [Whi82], and Brost [Bro88]. Brost is notable here because he applied the techniques of uncertainty to grasping.

When there is uncertainty in grasping, unpredictable, perhaps undesirable, results are possible. Brost research resulted in a general technique for planning parallel jaw grasping motions for arbitrary two-dimensional polygonal objects which need not be of uniform density. Brost desired to provide a set of motions that would guarantee a successful grasp of an object and avoid object *wedging*[4]. The structure of the problem assumes knowledge of only the shape of the object while the position and orientation of the object are all uncertainties. In addition, his system is robust even under gripper position and velocity errors.

A brief examination of Brost's technique, gives valuable insights into the grasping problem. Brost observed that if both fingers made simultaneous contact with the object and the object stayed in contact with both fingers throughout the motion, then object would either rotate as it was squeezed or it would wedge between the fingers, prohibiting further motion. From this result he constructs a *squeeze-grasp diagram* that essentially parameterized the squeezing operations that achieve a *stable grasp*[5] without getting stuck in an undesirable wedging condition. However, he noticed that for some objects, *no* squeeze-grasp operations resulted in a successful grasp.

He overcame this problem by insuring that one of the two fingers made contact with the object first. Brost observed that during a real grasping operation, there is generally a phase when only one finger is touching the object before both fingers make contact and during that time, a single finger pushes the object as it moves. This time he constructed a *push-stability diagram* that parameterized gripper motions into pushing operations by a single finger that resulted in known final configurations. Thus, after the pushing operation, two degrees of object uncertainty were removed. A stable grasp for the object was found by integrating the two operations into a *push-grasp diagram*. The final solution uses an *offset-grasp* where, rather than pushing and then squeezing the object, Brost only requires that the pushing finger touches the object first.

Two problems remained inherent in Brost's approach. His technique was limited to two finger parallel jaw grippers making it far from general purpose. However, his work is useful for manufacturing and automated assembly tasks where parallel jaws are more common. Secondly, there were still final grasping configurations of the object in the gripper that were not possible using this grasping strategy.

[4] An object is *wedged* when in its final configuration between two jaws none of the edges of the polygon are flat against a finger.

[5] Stable grasp means a force-closure grasp.

6 Synthesis of Multifingered Grasps

Research work done by Reuleaux [Reu75], Lakshminarayana [Lak78], and Mishra *et al.* [MSS87] proved minimum contact conditions for various grasps. Baker, Fortune, and Grosse [BFG85] proved that any two-dimensional polygonal object can be pretended stably with three fingers so that its weight is balanced. Additional work by several others showed methods for testing the stability of a grasp, namely if it was form/*force-closure*[6]. With the groundwork layed out, research turned towards the automatic synthesis of such stable grasps.

6.1 Early Robust Grasping

Early work in this area was done by Nguyen [Ngu88] in the late Eighties. Nguyen presented a fast and simple algorithm for directly constructing force-closure grasps based on the shape of the object. His construction is interesting because it does not generate a unique grasp but the complete set of all force-closure grasps on a set of edges and faces. He gets this result because he constructs each of the *independent regions of contact* for the fingers. The construction is exponential in the minimum number of required fingers and polynomial in the number of total fingers.

[6] A grasp is *force-closure* if given any external *wrench* applied to the object, there exist contact forces such that the force can be resisted.

Nguyen observed that two point contacts with friction at points P and Q form a planar force-closure grasp if and only if the segment PQ, or QP, points strictly out of and into the two friction cones of P and Q respectively. This important observation has been exploited throughout the grasping literature and research up to the present day. Nguyen's result can also be directly applied to the three-dimensional case with *soft-finger contacts*[7].

Like Brost, Nguyen approached the synthesis problem by insisting that grasps require as little accuracy as possible. He constructed two-dimensional grasping regions using two point contacts with friction by casting the problem into one of fitting a two-sided cone cutting the two contact edges into two segments of largest minimum length. Using similar techniques, he went on to describe algorithms for three-dimensional grasps using two soft-finger contacts, three-dimensional grasps using three *hard-finger contacts*[8], two-dimensional grasps using four frictionless contacts, and three-dimensional grasps using seven frictionless contacts. The complexity results were $O(n)$ time to analyze whether a grasp with n given contacts was force-closure and $O(n^c c 2^c)$ time to synthesize the n independent contact regions of an object requiring at least c contacts. The robust nature of Nguyen's solution using *independent contact regions* means that fingers can be placed independently at any position in the regions and still satisfy a force-closure grasp.

6.2 Other Grasp Synthesis Strategies

As mentioned in section 3, Mishra, Schwartz, and Sharir included a grasp synthesis section their paper [MSS87] for polygonal/polyhedral objects. Using point contacts without friction, they present a technique for constructing a grasp with $O(n)$ fingers in $O(n)$ time when n is the number of faces of the object and the dimension of the object is fixed.

The construction is a two step process. First they show how to synthesis a point contact grip using $O(n)$ fingers in $O(n)$ time. This result comes from the fact that any object with n faces can be held at equilibrium by n point contacts, one at each face. In he next step, they describe how to reduce this grip to another with the minimum number of fingers necessary for the grasp using portions of Steinitz's Theorem [Ste13].

The reduction step takes $O(n)$ time and involves choosing the correct coefficients to achieve a positive linear combination of a subset of the original contacts. They conclude with extending their construction technique to three dimensions yielding the same time complexities (for fixed d).

There are several problems with their technique. First, it is not useful in certain applications that require that the fingers be placed only in certain permissible regions of the object. Also, there are no limitations placed on the forces that the fingers can exert. This means that large forces may be required by the result of their algorithm that are not obtainable or are so large that they deform and/or break the object and/or gripper. Finally, it is not robust to the uncertainties involved with real world multifingered hands.

Markenscoff and Papadimitriou [MNP90] devote the majority of their work to showing the same results for minimum number of finger contacts for various grasps and contact types. However, their analysis does provide some new research in that it considers objects that include curved surfaces. In addition they show that for point contacts with friction, under the most relaxed assumptions three fingers are necessary and sufficient for force-closure in two-dimensions and four fingers in three-dimensions. They also give interesting construction techniques for generalized objects[9] including curved objects.

Their grasp analysis and construction centers upon the idea of maximal inscribed circles. They use these circles by choosing finger contact points either on or close to points of contact of a maximal inscribed circle with the boundary of the object. The use of maximal inscribed circles is not a new idea, in fact it was first utilized in a proof by Baker, Fortune, and Gross [BFG85] in the context of stable grasps (not form-closure) of polygons.

Throughout their analysis, they are forced to handle numerous special cases. The most common special case occurs when the maximal circle touches the boundary of the object at only two points. Their proof runs through many of these weird special cases and is hence not elegant.

For the three-dimensional case[10], maximal inscribed *spheres* are used for their constructive proofs just as maximal inscribed circles were used for the two-dimensional case. Again we note that curved surfaces are considered, making this approach quite general for any two- or three-dimensional object.

[7] Soft-finger contacts are contacts that can not only exert forces in any direction inside the friction cone of that object but also torques about the normal to the surface normal.

[8] *Hard finger* contacts are simply point contacts with friction. That is, they can exert forces in any direction that it within the friction cone for that contact.

[9] Object here means a closed, bounded, non-degenerate two dimensional body with a boundary that is connected and piecewise smooth. Smooth is defined here to mean continuous and having continuous derivatives of any order

[10] Three-dimensional objects are bounded, closed, non-degenerate, and with piecewise smooth, connected boundaries

When friction is considered, they show that two-dimensional form closure of any object (including the circle) can be achieved by three fingers (as opposed to four required without friction) and in three-dimensions, form-closure of any object (even with an axis of symmetry) is possible with four fingers (as opposed to seven). Since their proofs are all constructive in nature, they claim that the only difficult step in reformulating their work into a grasp synthesis solver would be to compute the maximal inscribed sphere of the object which they note can be carried out in linear time in the number of faces of the object using Megiddo's algorithm for linear programming [Meg80].

7 Early Optimal Grasping Techniques

Markenscoff and Papadimitriou [MP89] formulate and solve the problem of finding the optimal form-closure grasp of any given polygon. They optimize the grip with respect to minimizing the forces needed to balance the object's weight through friction. In essence they minimize the worst-case forces needed to balance any unit force acting on the center of gravity of the object. As we saw before, by minimizing the necessary forces on the object, we avoid requiring large forces that cause unnecessary stress and deformation on both the object and the robot hand. Also, there are often force limitations in our actual system due to the actuation of the joints in the hand, making this a natural constraint.

They first present an analytical method that uses the metric from above to calculate the quality of a grip of a polygonal object by three fingers. In the second part of their paper, they attack the more difficult problem of optimizing the form-closure of a polygon which they do by solving a sequence of reductions using linear programming duality and some intricate plane geometry. This leads to an algorithm, that can rapidly analyze any reasonably complex object and compute the optimum grip by examining a finite number of cases (growing as the cube of the number of sides of the polygon).

In their analysis they run into special cases similar to those in [MNP90]. For two of those cases, concave vertices and parallel sides, they give special procedures for computing the optimum grip of a polygon with three fingers. Although their results are more general than Nguyen's [Ngu88], the necessary consideration of special cases, make this a somewhat messy solution.

8 Robust Grasp Planning

8.1 2D Polygons with 3 Fingers

Ponce and Faverjon obtained excellent results using a new approach to constructing three-finger force-closure grasps of polygonal objects [PF91]. They consider a hand equipped with three hard-finger contacts from which they prove a sufficient condition for force-closure grasps leading to a system of linear constraints in the position of the fingers along the polygonal edges. Like Nguyen [Ngu88] for two finger grasps, they determine maximal segments of the object boundary where the fingers can be positioned independently while maintaining force-closure.

The regions satisfying these constraints are found by a projection algorithm based on linear parameter elimination accelerated by simplex techniques. They solve this problem of projecting linear convexes in n-dimensions along one of several coordinate axis directions with linear programming, namely the simplex algorithm. The search for a feasible vertex in the original convex has to be done only once for all hyperplanes, thus making the simplex algorithm efficient.

Once the contact regions are found they use a reasonable criterion to maximize the minimum of the lengths of the three contact regions. However, in general there is not a unique solution to this problem, as for sufficiently long edges, the size of the contact regions depends only on the size of the friction cones. They choose an additional criterion that attempts to place the center of mass of the object at the center of the friction cones. This enables them to decrease the effect of gravity and internal forces during the motion of the robot.

As we hinted at previously, the central idea behind their synthesis algorithm is that they translate the problem of finding maximal independent contact regions into a linear programming problem with ten variables that can be solved using the simplex algorithm. The function to maximize becomes simply a weighted combination of the two criteria: (1) the minimum of the lengths of the three intervals and (2) the the position of the center of gravity in the friction cones (at the center is maximal, hence optimal). A typical result from this technique is shown in Figure 2.

In principle their approach can be extended to more complicated problems such as the synthesis of three- or four-finger grasps of three-dimensional objects. However, linear sufficient conditions are difficult to find for these cases, and the number of variables and equations involved make solving these problems much more difficult.

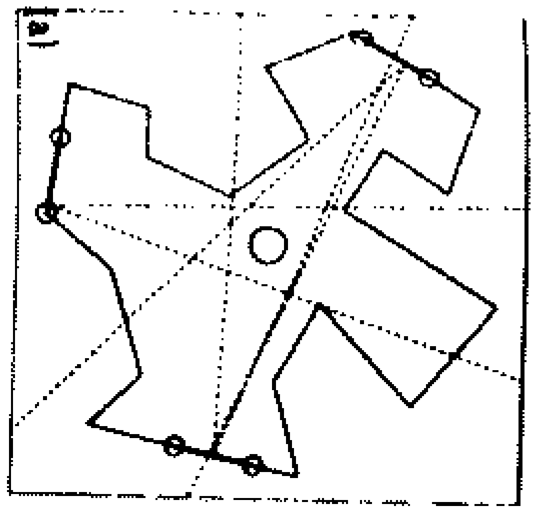

Figure 2: An object and its best grasp using the techniques from Ponce and Faverjon in 1991. The maximal independent contact regions are shown as heavy lines with security margin circles at the ends of the regions.

8.2 2D Curved Objects with 2 Fingers

Faverjon and Ponce returned to the issue of computing force-closure grasps in [FP91], considering the problem of grasps on *curved* objects in two-dimensions using two-fingers with friction. The objects that they considered were modeled by parametric curves, and the force-closure grasps characterized by a systems of polynomial constraints in the parameters of these curves. All configuration space regions satisfying these constraints were found by a numerical cell decomposition algorithm based on curve tracing and continuation techniques.

As before, maximal object segments where fingers can be positioned independently were found by optimization within those grasp regions. This was an extension of Nguyen's work and like Nguyen, they characterize two-finger force-closure grasps by the fact that the line joining the contact points must lie within the friction cones at those points.

As we have seen before in [PF91] and [Ngu88], the linear form of the force-closure constraints in the polygonal case allowed for the design of fast and simple geometric algorithms for the constructing stable grasps. However, for smooth (and piecewise-smooth) curved objects, these constraints become highly nonlinear, calling for an approach of more algebraic nature to grasp synthesis.

They begin their analysis by modeling the boundary of the planar object to be grasped by a piecewise-smooth collection of parametric curves. Each of the two contact points, lie on a parametric curve given by $(x_i(u_i), y_i(u_i))$ for $i = 1, 2$ and x_i and y_i are polynomials in u_i. For a given pair of curve segments, a grasp is completely determined by a pair (u_1, u_2). This pair takes values in the grasp configuration space defined by the intervals of u_1 and u_2. They then re-write the friction cone conditions for each contact point for force-closure in algebraic terms. They observe that in the grasp configuration space the force-closure grasp regions are bounded by sets of algebraic curves. To characterize the topology of the regions and their bounding curves they use a numerical cell decomposition algorithm, whose output is a description of the stable grasp regions, their bounding curves, and their adjacency relationships.

During this step of the construction the most expensive part of the algorithm is the computation of the extremal points and intersection points of the grasp configuration space curves. Finding those features amounts to solving square systems of polynomial equations. For this they use the continuation method, a global numerical technique that can find all solutions of systems of polynomial equations having up to a few thousand roots using [Mor87].

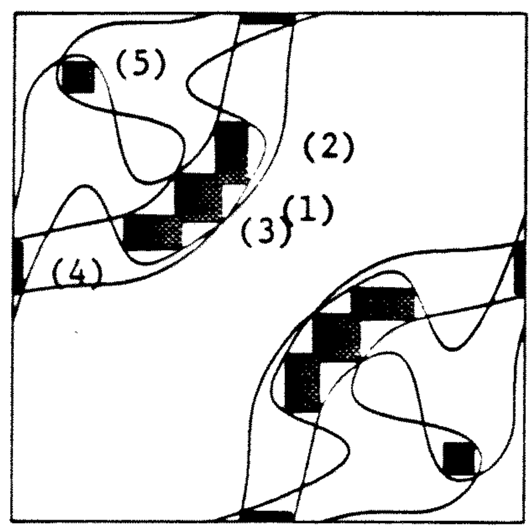

Figure 3: The maximal rectangles found inside every valid region in the grasp space from work by Faverjon and Ponce in 1991.

They next obtain maximal grasp rectangles with sides parallel to the coordinate axes since it is these rectangles that realize the local maximum size of the grasp regions as desired. For each rectangle, they decompose the four variables (the edges of the rectangle) into two embedded optimization process in two variables. The resulting maximal area rectangle will have the maximal segments on the objects boundary where the fingers can be positioned independently while maintaining force-closure. This results in a more robust grasp planner, making implementation for a real robot with uncertainties possible. Typical results from this algorithm are shown in Figures 3–4.

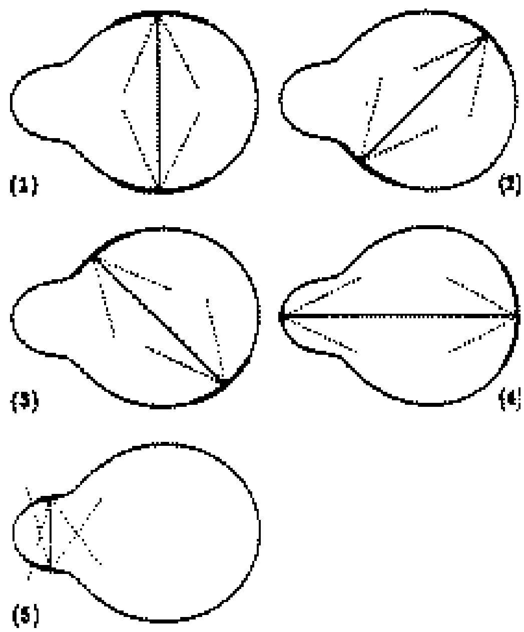

Figure 4: The five grasps corresponding to the rectangles of the previous figure.

This method by Faverjon and Ponce is not without problems and areas of possible improvement. One important area that is incomplete is in determining a bound on the number of maximal rectangles within each stable grasp region. This in turn would provide better methods for generating starting points for the corresponding optimization process since it would minimize the number of initial guesses. In addition, although their technique works well in practice, it has not been proven to yield all local maximal rectangles, making it incomplete. However, their approach is quite elegant and general. In principle, it can be extended to more complicated problems such as the synthesis of three (or more) finger grasps of possibly three-dimensional objects. However, as before, the number and degree of the equations involved make such an implementation difficult.

8.3 3D Polyhedra with 3 and 4 Fingers

Ponce, Sullivan, Boissonnat, and Merlet [PSBM93] extend the approach of [Ngu88], [PF91], and [FP91] to the synthesis of three- and four-finger force-closure grasps of three-dimensional polyhedral objects again using independent maximal grasp regions. They show that for polyhedral objects, the sufficient conditions for force-closure are linear in the unknown parameters which reduces the problem of computing force-closure grasps of polyhedral objects to the problem of projecting a polytope onto some linear subspace.

Their research uses the fact that a necessary condition for three points to form a force-closure grasp is that there exists a point in the intersection of the plane formed by the three points with the double-sided friction cone at these points. For four finger grasps they derive results that will allow a stronger linear sufficient condition for three-finger force-closure grasps in the case where gravity acts as a fourth finger. In the four finger case, they utilize the fact that a sufficient condition for four-finger contact to be force-closure is that there exists a point in the intersection of the four open internal friction cones with the tetrahedron formed by these points.

The sufficient conditions of force-closure can be written as a set of linear inequalities in the position of a point x_0 and the position of three or four contact points x_i. For polyhedral objects, each contact can be parameterized linearly by two variables (u_i, v_i) which specify its position in the plane of the corresponding face. To obtain all the solutions (u_i, v_i) yielding force-closure grasps, they show that they must eliminate the point x_0. This results in a projection problem. The problem is to construct the orthogonal projection of the d-polytope onto a k-dimensional subspace of the original d-dimensional Euclidean space. A simple implementation of this projection algorithm takes $O(tn)$ time where t is the size of the projection of the d-polytope and n is the number of intersecting half-spaces used to define the polytope. They show that this time complexity can be improved by preprocessing the hyperplanes using a recent result by Matoušek and Schwarzkopf [MS92]. In general they show that the projection algorithm for a d-polytope onto a k-dimensional subspace can be computed in time and space of $O(kn^{\gamma+\epsilon}t^{\gamma+\epsilon})$ where ϵ is any positive constant, t is the size of the projection of the original d-polytope, and (roughly) $\frac{2}{3} \leq \gamma \leq \frac{3}{4}$.

Again because of uncertainties in robotics systems they attempt to minimize the sensitivity of a grasp to positioning errors. They achieve this by finding triples of independent contact regions as. These regions, as we saw before, are such that for any tuple of contact points chosen in them, the corresponding grasp is force-closure. From before, we know that finding maximal independent contact regions reduces to solving a linear programming problem, making their overall solution quite nice.

9 Optimal Grasps

Ferrari and Canny [FC92] address the problem of computing optimal force-closure grasps of polygonal objects by computing a maximal ball included in the convex hull of the contact *wrenches*. Their notion of an optimal grasp, called *quality criteria*, actually focuses around two criteria that consider the total finger force and the maximum finger force. They define the magnitude of the wrench as $||w|| = \sqrt{||F||^2 + \lambda||\tau||^2}$ with the choice for λ being somewhat arbitrary. The first of the *quality criteria* is concerned with finding the grasp configurations that maximize the wrench, given independent force limits (i.e. that minimize the worst-case force applied at any point contact). The second criteria minimizes the sum of all applied forces. Since the magnitude of the force is proportional to the total current in the motors and amplifiers, this second criteria results in the minimization of the power needed to actuate the gripper.

Their major insight is that in the wrench space, the convex hull of the finger wrenches on the object have important geometrical interpretations. Namely, that their *quality measure* is simply the distance of the nearest facet of the convex hull from the origin. This means that optimal contacts can be synthesized in the wrench space by simply computing the convex hull of candidate contacts, determining the facet of minimum distance from the origin in the wrench space, and choosing the maximum of these distances. This has to be repeated for each side of a polygon where the number of possible configurations grows linearly as the number of edges of the polygons while the computation of the convex hull of the primitive wrenches in the wrench space takes constant time. This results in an $O(n)$ time algorithm with n the size of the polygon. Essentially the quality criteria of the grasp is given by the value of the radius of the largest closed ball centered in the origin of the wrench space, contained in the set of all the possible wrenches that can be resisted by applying at most unit forces at the contact points. Their solution can also be applied to any three-dimensional object.

The advantage of this solution is that these *quality criteria* can be easily calculated for a wide variety of objects/parts in hopes of feeding these quality values to a planner that will choose optimal tool/gripper selection for each object/part in an assembly task. The motivation for using different grippers, each of which is suitable for a small subset of operations, along with an automated tool-changer, is the philosophy of RISC Robotics.

RISC Robotics (Reduced Intricacy in Sensing and Control) is a class of robotics that relies on simple control and sensory information. Sensory information in a RISC Robotics system (1) require little or no processing time (typically single bit digital I/O signals are used), (2) is inexpensive (thus making redundancy a feasible option), and (3) is highly accurate (typically on the order of 70 microns). RISC Robotics follows many of the same paradigms of RISC for computer architecture by using simple tools to build up and perform more complicated tasks. In terms of grasping, RISC Robotics moves away from multifingered hands because of their intricate design making them expensive, unreliable, computationally heavyweight, and difficult to control and create planners for. In general, a RISC Robotics system maintains a small set of simple, inexpensive, and accurate machines that can be easily interchanged and/or combined to perform more complex tasks.

10 Future and Current Work

Exciting new work is ongoing into many of the non-holonomic constraints in grasping. Recent work by Li, Murray, and Sastry [LMS93] examines rolling finger contacts and object sliding constraints. Related work in a recent paper by Chen and Burdick [CB93] investigates finger gates on planar objects. Essentially, most of the past grasping research was for static grasps, whereas new work looks at various motions that can be done once an object is being held and exactly how to plan and control such operations. Some of the motivation for this work comes from the observation that the grasp used for picking up a pencil is entirely different from the one we use for writing, even though the geometry remains the same.

There is also still exciting work ongoing into grasp planning, in particular planning optimal grasps. Most of the work in this areas that was presented in this paper have come out of research within the past year. In addition, work into formalizing a synthesis technique to accompany the 1992 paper [FC92] is in process by Mirtich and Canny.

11 Conclusion

We have also seen that manipulation is complex, typically involving combinations of open and closed loop kinematic chains, non-holonomic constraints, redundant degrees of freedom, and singularities. In addition their are non-linearities in the contact conditions between soft fingers and grasped objects as well as the actuator dynamics. As a result, many assumptions had to be made to make the problem tractable. Yet, humans can perform such tasks easily suggesting that a simple approach exists.

In addition, an interesting insight made by

Nguyen [Ngu88] and others, observed that larger friction cones were produced when finger contacts were at edges or vertices of an object. This explains why people grasp objects at the edges and corners and why the contacting surface of human fingers need to be soft rather than hard like the fingernails. However, this fact has yet to be properly addressed and exploited in the grasping community.

Finally, research pointed out by Ferrari and Canny [FC92] as well as others, moves grasping towards a new direction. This new research centers around the recent movement away from anthropomorphic hands towards specialized grippers and toolchangers. Such specialized tools are simpler in terms of kinematics and controllability, and far more accurate than a human like hands. After all, the hand has evolved over millions of years as an organ used as much for sensation and communication as for manipulation. In fact, for many manufacturing tasks the human hand is less than ideal. When a mechanic starts to work on a machine, the first thing he reaches for is his/her toolbox, with pliers, wrenches, tweezers and work gloves to help him/her finish the job.

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